

Approximating 3D points with cylindrical segments

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1. Background

- Motivation: human behavior can be indicated by the movement of neurons, each of which is of a tree-like structure.
- General problem: Given a set Q of n points in 3D, compute a minimum set of minimal cylindrical segments S_1, S_2, \dots, S_{k^*} (with radii r_1, r_2, \dots, r_{k^*}) enclosing Q .

Extra parameters to conform with practical constraints.

- We should minimize the volume of the union of S_1, S_2, \dots, S_{k^*} .
- S_i should not ‘penetrate’ S_j after intersecting S_j .

- This problem is different from the problem of covering points with minimum cylinders. In fact, when $k > 1$, given the same input the solutions for these two problems might be different.

- Practical solution: Computational biologists use commercial software to reconstruct a polyhedron (or a set of polyhedra), then estimate the problem manually with cylindrical segments.
- Our objective is to help them automating this process.

- Complexity of the problem: Strongly NP-hard. Covering 3D points with minimum number of line segments is strongly NP-hard, which implies that any version of the problem is strongly NP-hard as long as the objective function $f(r_1, r_2, \dots, r_{k^*})$ is monotone [Zhu'2002]. (r_1, r_2, \dots, r_{k^*} are the radii for S_1, S_2, \dots, S_{k^*} .)
- Status of the problem: If the number of cylindrical segments is a fixed k , then there is a PTAS for any fixed k when the objective function $f(r_1, r_2, \dots, r_{k^*})$ is monotone (i.e, sum of radii, sum of volume) [Zhu'2002].
- **Reference:** [Zhu'2002] B. Zhu. Approximating 3D points with cylindrical segments. Proc. 8th International Computing and Combinatoric Conference (COCOON'2002), Singapore, Aug, 2002 (to appear).

2. Algorithmic Improvements (May, 2002—)

- Zhu's PTAS for fixed k is based on the following factor-2 approximation for the problem when $k = 1$.
 - Step 1. Compute the diameter $D(Q)$ of Q . Let $D(Q)$ be $d(p_1, p_2)$.
 - Step 2. Use p_1p_2 as the center of the approximating cylindrical segment A . The maximum distance between point $q \in Q$ and p_1p_2 is the radius of A . (Also, return the length of A , $d(p_1, p_2)$.)
- The above algorithm runs in $O(n \log n)$ time and provides a factor-2 approximation for the problem when $k = 1$.
- With simple modification this algorithm can be implemented as a $(1 + \delta)$ -approximation for the problem when $k = 1$, with running time $O(n \log n + n/\delta^4)$ which is the basis of the PTAS for fixed k .

- In practice, computing the diameter of a set of 3D points optimally is not easy. We recently propose the following algorithmic variation.
 - Step 1. Pick any point $p \in Q$ and compute the farthest point q from p .
 - Step 2. Use pq as the center of the approximating cylindrical segment A' . The maximum distance between point $r \in Q$ and pq is the radius of A' .
- The above algorithm runs in $O(n)$ time and provides a factor-4 approximation for the problem when $k = 1$.
- With simple modification this algorithm can be implemented as a $(1 + \delta)$ -approximation for the problem when $k = 1$, with running time $O(n/\delta^4)$; moreover, this algorithm is elementary and does not need any complex subroutine.

3. Implementation considerations

- Notice that Zhu's PTAS for fixed k runs in $O(n^{3k-2}/\epsilon^{4k})$ time, so it is hardly useful when $k > 2$. Informally, this explains why biologists use reconstructed polyhedra (instead of sample points) as input.
- Possible input: images, sample points and reconstructed polyhedra.

- For the first stage of implementation (May, 2002 — September, 2002), we will use polyhedra as input, i.e., given a polyhedron (set of polyhedra) P , try to fit P with ‘best’ cylindrical segments.
- So far, we have a method to identify critical edges of P , i.e., those edges around which the polyhedron encounter great structure change (making a turn, etc).
- Students involved: Wenhao LIN and Xun HE.

4. Problems

- Theoretical Problems:
 - (1) For $k = 1$, can we have a simple $1 + \epsilon$ approximation which runs in $O(n/\epsilon)$ time? Best known algorithm achieves this bound [Chan'2000], but is too complex. The one used in [Zhu'2002] is simple, but runs in $O(n/\epsilon^4)$ time. So far we have another simple $1 + \epsilon$ approximation which runs in $O(n/\epsilon^2)$ time — but in practice we believe it runs in $O(n/\epsilon)$ time. More analysis/simulation is needed.
 - (2) If we can't solve (1) in satisfactory, can we try the same problem, but the input is a convex polyhedron with n vertices?
 - (3) Can we improve the running time of Zhu's PTAS for fixed k ?
 - (4) Can we obtain a PTAS for arbitrary k ?
 - (5) Can we obtain a PTAS for fixed k when the objective

function is the volume of the union of S_1, S_2, \dots, S_{k^*} ?

- Practical Problems:

- (6) When the input is a polyhedron P (or a set of polyhedra), how can we break P at right places?
- (7) When user interference is allowed (i.e., some expert can see that at certain places the reconstruction software make some mistake), how can our software incorporate this user input to have a better solution?